Remarks by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

In the second term of the second member of the equation, $\frac{1}{4}(p-4)$ should be $\frac{1}{4}(p-3)$. The equation is generally but not universally true as is shown by induction in what follows.

Let $p=4m+3$, then $\frac{1}{2}(p-1)=2m+1$, $\frac{1}{4}(p-3)=m$,

$$\sum_{n=1}^{\frac{1}{2}(p-1)}\left[\frac{n^2}{p}\right]=\frac{p-3}{4}\cdot\frac{p-1}{2}-\sum_{n=1}^{\frac{1}{4}(p-3)}[\sqrt{(np)},$$

or as follows:

$$\begin{array}{rr}
 & A=\ B-C, \\
m=2, & 3=\ 10-7, \\
m=3, & 7=\ 21-14, \\
m=4, & 11=\ 36-25, \\
m=5, & 18=\ 55-37, \\
m=7, & 34=105-71.
\end{array}$$

If one of the $[n^2/p]$ is an exact quotient, and hence one of the $[\sqrt{(np)}]$ rational, the equation is $A=1+B-C$.

$$m=6,\ p=27,\ [9^2/p]=3,\ \sqrt{(3\times27)}=9,$$
$$m=15,\ p=63,\ 21^2/p=7,\ \sqrt{(7\times63)}=21.$$

$\therefore A=1+B-C$, $m=6...25=1+78-54$, $m=15...153=1+465-313$.

If two of the $[n^2/p]$ are exact quotients, and hence two of the $[\sqrt{(np)}]$ rational, the equation becomes $A=2+B-C$.

$$m=18,\ p=75,\ 15^2/p=3,\ \sqrt{(3\times p)}=15,\ 30^2/p=12,\ \sqrt{(12\times p)}=30.$$

$\therefore A=2+B-C$ becomes $219=2+666-449$ for $m=18$. $A=t+B-C$ is the true universal equation.

The geometric proof in this solution is wanting. Who can produce it? ED. F.

155. Proposed by PROF. R. D. CARMICHAEL, Anniston, Alabama.

If p and q are primes and m and n are any integers, find the cases in which the equation $p^m-q^n=1$ may be satisfied.

Remarks by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Some values, found by inspection, are given in the following table:

p	q	m	n
3	2	1	1
3	2	2	3
2	1	1	1
2	31	5	1
2	127	7	1

ERRATUM. In 156 for e^2 read e^3.

AVERAGE AND PROBABILITY.

196. Proposed by R. D. CARMICHAEL, Anniston, Ala.

A circle is inscribed in a square. Find the chance that the distance between two points within the square and without the circle shall not exceed a side of the square.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $2a$=side of square; (x, y), (u, v) the coordinates of the points; $\sqrt{(a^2-x^2)}=y_1$, $\sqrt{(a^2-u^2)}=v_1$. If both points are in the same corner, the distance between them is always less than $2a$. If both points are situated one each in opposite corners, the distance between them is always greater than $2a$. If both points are placed one each in adjacent corners, we have $\sqrt{[(x-u)^2+(y+v)^2]}=4a^2$, for the greatest distance between the points and $\sqrt{\{(x-u)^2+[\sqrt{(a^2-x^2)}+\sqrt{(a^2-u^2)}]^2\}}=\sqrt{[(x-u)^2+(y+v)^2]}$, for the least distance between the points.

$\therefore v_2=\sqrt{[4a^2-(x-u)^2]}-y$, $v_3=\sqrt{(a^2-x^2)}+\sqrt{(a^2-u^2)}-y$.

$$\therefore p=\frac{\int_0^a\int_0^a\left[\int_{y_1}^a\int_{v_1}^a dydv+\int_{y_1}^a\int_{v_1}^{v_2}dy\,dv\right]dxdu+\int_0^a\int_0^x\int_{y_1}^a\int_{v_3}^{v_2}dxdudydv}{3\int_0^a\int_0^a\int_{y_1}^a\int_{v_1}^a dx\,dy\,du\,dv}$$

$$=\frac{a^4(4-\pi)^2+0+.16\int_0^a\int_0^x\int_{y_1}^a\int_{v_3}^{v_2}dx\,du\,dy\,dv}{3a^4(4-\pi)^2}=\tfrac{1}{3}+\frac{1.6}{3a^4(4-\pi)^2}.M.$$

$$M=\int_0^a\int_0^x\int_{y_1}^a\{\sqrt{[4a^2-(x-u)^2]}-\sqrt{[a^2-x^2]}-\sqrt{[a^2-u^2]}\}dx\,du\,dy$$

$$=\int_0^a\int_0^x[a-\sqrt{(a^2-x^2)}]\{\sqrt{[4a^2-(x-u)^2]}-\sqrt{[a^2-x^2]}-\sqrt{[a^2-u^2]}\}dx\,du$$